The Tile Puzzle: An Experiential Exercise in Introductory Management Science

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Teaching management science requires presentation of concepts that are both interdependent and abstract. This teaching brief describes a reoccurring class exercise, the Tile Puzzle, intended to support a conceptual sequence of linear programming formulation, solution methods, and sensitivity analysis. Designed for intermittent use, the Tile Puzzle provides an underlying “hands-on” tangible model threading together a series of critical concepts over several class sessions. The physical materials required to stage this in-class exercise are compact and inexpensive, making the Tile Puzzle both highly portable and feasible for use in the context of very large classes.

**Keywords:** in-class exercises, linear programming, group activities

One challenge of teaching introductory management science is the requirement that students master a sequence of concepts that are both interdependent and rather abstract. The powerful benefit of concrete examples in the comprehension of abstract concepts has long been known to designers of instructional materials (Morrison, Ross & Kemp, 2004). Many successful in-class exercises in college business courses support learning by providing participants with a tangible representation of a broader concept, such as the use of Tinker Toys to represent a discrete part assembly process, as suggested by Heineke and Meile (1995). Presented in this teaching brief is The Tile Puzzle, a highly successful
in-class exercise providing this same assistance, but simultaneously supporting the following three additional educational objectives:

1. Repeated use to manage step size: Once introduced, the Tile Puzzle can be “used” more than once over several class sessions, including serving as the subject of two different in-class competitions. Use of consistent terminology and explicit reference to earlier material are two known successful strategies for managing step size, or the transition between ideas in an instructional sequence (Morrison, Ross & Kemp, 2004). The reoccurring Tile Puzzle supports this in the context of linear programming, creating a single “thread” through a series of abstract ideas.

2. Blending with lecture: The Tile Puzzle is designed with “intervening class content,” dropping the tangible problem in favor of a return to abstract concepts to move the progression forward. Research on instructional materials suggests that the mixing of abstract and concrete information may be the most beneficial to the learner (Sadoski, Goetz & Fritz, 1993).

3. Low cost, high portability: Unlike in-class exercises that require supporting elements such as children’s toys or game pieces, the physical model of the Tile Puzzle can be readily recreated with brightly colored paper, as seen in Figure 1. These paper pieces represent the limited resources in an linear programming problem first introduced to participants as the challenge described in Appendix A. The choice of these materials, versus more elaborate tangible props, eliminates the cost and logistical barriers to staging such an exercise in the context of an extremely large class. Technical advice for the creation of the supporting “tile packs” is included in Appendix B of this document.

The sequence of exercises that comprise the Tile Puzzle has been in use since 1998 in an Executive MBA course titled Introduction to Management Science. In this setting, 30–40 students have been divided into 5–8 study groups, and these groups become the participants in the
Figure 1. Paper Tiles Used in Tile Puzzle Exercises.
exercises. However, these same exercises have been used on several occasions to support a linear programming module within an undergraduate operations management class with 120–160 students attending. The inexpensive nature of the physical model itself, packs of brightly colored paper tiles, makes deployment of the exercise feasible in the presence of 15 or more groups.

**Instructional Design of the Tile Puzzle**

The Tile Puzzle as it is introduced to the learner is stated explicitly in Appendix A, and is essentially a combinatorial optimization problem. The following sequence of activities is intended to begin on the first day of management science, be that literally the first day of a management science class, or the first day of a management science module housed within an operations management class.

**First Occurrence**

Class Competition I: Students are presented with the puzzle displayed in Appendix A, as well as issued the physical model of that problem represented by paper “tile packs”. Everyone is invited to address the question posed at the bottom of the page, “What should you do?” Often, the initial response will be, “make profit.” When asked, “How?” students will then volunteer some variation of, “by assembling the patterns” and are often puzzled why this needs to be stated. This sets up the formal introduction of decision variables versus objective functions, which are frequently confused during first attempts at formulation. To underscore this distinction and to motivate the competition, the discussion leader can draw a table of five columns on the classroom board, four of the columns representing potential solution values and the fifth column the solution’s associated profit. The discussion leader then proposes an answer to the question, “What should you do?” posed at the bottom of the Appendix A: “Do nothing.” This becomes the first entry in the table, as a series of zero values for patterns assembled and profit. When challenged to find
a reason why the proposed solution cannot be implemented, students find that such a reason doesn’t exist with respect to the language in the problem. At this point, the term feasibility can be introduced: while students’ intuition tells them that the discussion leader’s solution is likely an extremely poor one with respect to the logical objective of the problem, it is nonetheless a feasible solution. At this point, the students are given a 15–20 minute work period to manipulate the problem within their groups, to identify a better solution.

At the end of this work period, the student groups are called upon to each announce their findings. Groups will often adopt different styles when tackling the problem, which can be drawn out as the solution table is completed. Some groups will decline to manipulate the physical model, relying on calculations to develop a solution. It is not uncommon for such a group to inadvertently create a highly profitable, yet infeasible solution. Typically, at least one group has devised the solution, “no valleys, five mountain peaks, three butterflies, and two water lilies,” with an associated profit of $430. This can be highlighted as the result of a greedy algorithm, stressing that the term is not a criticism of the group’s approach, but the standard description of a solution strategy which focuses on maximizing overall benefit by maximizing the most beneficial element. (The mountain peak pattern is the most profitable of the mix, at $70 per pattern.) Some groups will develop profitable solutions with the assumption that they must produce at least one of each pattern, but almost always at least one group will volunteer, “eight valleys, one mountain peak, five butterflies and no water lilies,” with an associated profit of $560. No other group will best this, in that this is the optimal solution to the Tile Puzzle. In discussing the “win”, it is beneficial to point out that the champion solution included only one representative of the most profitable pattern, and the second-most heavily stressed pattern was simultaneously the weakest in terms of profit (butterflies clear only $10 per pattern assembled). Furthermore, while there is reason to suspect that this is the optimal solution, it cannot be declared optimal at this point.
Second Occurrence, Introduction to Algebraic Formulation

Immediately following the competition, the Tile Puzzle becomes a natural first lecture example of algebraic formulation. It is imperative that this exercise follows the competition immediately, while memory of manipulating the tiles is fresh and the paper tiles are still in evidence. Under these conditions, even students uncomfortable with mathematics can often “see” the process of translating the objective and limitations of the Tile Puzzle into algebraic expressions. Figure 2 displays one finished presentation of that formulation.

Intervening Class Content

Now it is helpful, although not mandatory, to formulate additional problems unrelated to the Tile Puzzle, to strengthen student confidence in formulation. Students can then be introduced to solution methods such as graphical analysis and the use of optimization software such as Solver or What’s Best! in Excel, which is then deployed to confirm the optimal solutions.

Third Occurrence: Solving the Tile Puzzle as a Spreadsheet

Having explicitly formulated the Tile Puzzle, and having been introduced to an optimization package such as Solver, students generally have little trouble restating the Tile Puzzle in a Spreadsheet. Once Solver is launched in this class demonstration, it will immediately return the optimal solution of “eight valleys, one mountain peak, five butterflies and no water lilies,” displayed in Figure 3. As mentioned previously, there is usually at least one group that identifies that solution during the course of the competition, and they are generally quite pleased with themselves to see confirmation of its optimality in class.

Truthfully, what has been solved here is the LP relaxation of the Tile Puzzle, which happens to be an integer solution. If integer requirements and related computational concerns have not yet been introduced at this point in the course, it is preferential to save mention of this until the issue inflicts itself on the final occurrence of the Tile Puzzle.
**Intervening Class Content**

Now various concepts associated with optimality can be discussed, such as binding versus non-binding constraints, and slack versus surplus. The paper-based Tile Puzzle can provide a vivid demonstration of the general concept of slack in a constraint when the instructor invites the students...
Figure 3. The Optimal Solution to the Tile Puzzle, as Displayed on a Spreadsheet.
to assemble the optimal solution stated in Figure 3, and consider the scene before them. Students will find that, having maximized profit, they nonetheless have two red triangles, two brown diamonds, and four white squares remaining unused, a tangible representation of the slack in these constraints. Students who found the optimal solution during the first class competition may now recall a certain frustration working with blue triangles, yellow triangles and/or green diamonds, and see now how the conditions of zero slack and binding are linked for a constraint.

**Fourth Occurrence: Class Competition II**

Tile packs are distributed again, and the groups are encouraged to assemble the optimal solution. The discussion leader then announces that additional tiles of *one, and only one*, type may be purchased for $5 more than the original tile cost, a surcharge levied by the instructor for “expediting”. The groups are given 15–20 minutes to determine which type and what quantity of tile they desire to purchase under these terms, to create the most profitable solution in these circumstances. These tiles are retrieved from the instructor, the transactions recorded, and then each group is polled for their revised solution. The most successful groups will have elected to purchase two blue triangles, to create an additional mountain peak pattern and increase profit by $60, inclusive of the surcharge on the additional triangles.

**Intervening Class Content**

The objective of the opportunity in the second competition is explained: the offer of only one tile type forced each group to consider which of the six constraints would be most beneficial to revise. The definition of *shadow price* can now be introduced, as well as its associated concept of *range of feasibility*.

**Fifth Occurrence: Solver Sensitivity Report for the Tile Puzzle**

At this point in class, the sensitivity report feature of optimization engines such as *Solver* can be demonstrated, generating a report for the Tile
Figure 4. Tile Puzzle Sensitivity Analysis Report, as Generated by Solver in Microsoft Excel.

Figure 4 displays an example of such a report, generated from the spreadsheet model visible in Figure 3. The success of the group(s) that won the second competition is confirmed, as the most profitable answer to the opportunity presented is the acquisition of two blue triangles. Some groups will have elected to experiment with additional yellow triangles or green diamonds, but their inability to dominate the competition is now reflected in the relative magnitude of the associated shadow prices. However, student interest in these tile types relative to the red triangles, brown diamonds or white squares, can now be validated by the fact that these represent constraints binding on the optimal solution, so only these shapes are associated with any potential improvement in profit.
A final application of the Tile Puzzle occurs naturally at the end of the discussion of shadow prices. Students are asked to recall the textbook definition of shadow price: the increase in the objective function value if the right-hand side of a constraint is increased by one unit. From the Sensitivity Analysis report, it then follows that profitability could be increased by $35 if one additional blue triangle tile were added to the resource base. However, what exactly is one supposed to do with just one additional blue triangle? Students consider the physical tile packs, but generally have no suggestions. Returning to the spreadsheet model, the discussion leader can manually increase the original right-hand side value of 10 to 11, and solve the revised spreadsheet again. A new solution yielding exactly $35 more profit appears: assemble 8 valleys, 1.5 mountain peaks and 5 butterflies. In the management science class, this becomes the launching point into the topic of integer requirements.
APPENDIX A

TILE PUZZLE AS SEEN BY STUDENT

Suppose you can create four different objects by arranging tiles of various shapes and colors:

Object 1, “The Valley”  
Object 2, “The Mountain Peak”

Object 3, “Butterfly”  
Object 4, “Water Lily”

Each “Valley” you assemble earns you $70, each “Mountain Peak” earns you $100, each “Butterfly” earns you $60, and each “Water Lily” earns you $55. However, you must pay for the tiles you use. Triangles, regardless of size or color, cost $5 each. Likewise, diamonds and squares cost $10 each. There are a limited number of tiles available:

<table>
<thead>
<tr>
<th>Tile Type</th>
<th>Total Number Available for Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blue Triangle</td>
<td>10</td>
</tr>
<tr>
<td>Red Triangle</td>
<td>12</td>
</tr>
<tr>
<td>Yellow Triangle</td>
<td>8</td>
</tr>
<tr>
<td>Green Diamond</td>
<td>10</td>
</tr>
<tr>
<td>Brown Diamond</td>
<td>12</td>
</tr>
<tr>
<td>White Square</td>
<td>10</td>
</tr>
</tbody>
</table>

WHAT SHOULD YOU DO?
APPENDIX B
TECHNICAL NOTE ON TILE PACK PRODUCTION

The tile packets that model the Tile Puzzle here are easily created from brightly colored card stock. Grid the shapes on graph paper, spray with temporary photo mount adhesive, and layer the graph paper onto brightly colored card stock. Cut according to the shapes drawn on the graph paper and peel graph paper from each tile when finished. To create tiles of the same dimensions as those appearing in Figure 1, cut blue triangles from 1” squares by halving the squares diagonally, whereas cut yellow and red triangles by cutting across both diagonals of a 1” square. White squares and both colors of diamonds should then be 11/16” on a side. Plastic bags work well to bundle the appropriate mix of tiles into individual tile packs. One author here manufactured fifty complete packs in one long afternoon in 1997, and these tile packs have been in popular service since.
REFERENCES

